PERFORMANCE ANALYSIS OF IMAGE COMPRESSION USING DISCRETE WAVELET TRANSFORM FOR DIFFERENT WAVELETS

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Abstract: Image compression is one of the major image processing techniques that are widely used in medical, automotive, consumer and military applications. Discrete wavelet transforms is the most popular transformation technique adopted for image compression. In this work a DWT based image compression technique is implemented. Input image is processed using DWT and IDWT algorithm and image compression is acheived. Biorthogonal, Haar and DB2 wavelets have been used to compare the performances of DWT-IDWT. MSE, CR and PSNR are the three parameters that are used to analyze the DWT performances and it is found that DB2 wavelet achieves better compression and also reconstruction is better in terms of MSE and PSNR.

1 Introduction

The discrete wavelet transform (DWT) is being increasingly used for image coding. This is because the DWT can decompose the signals into different sub-bands with both time and frequency information and facilitate to arrive a high compression ratio and it supports features like progressive image transmission (by quality, by resolution), ease of compressed image manipulation, region of interest coding. In wavelet transforms, we divide the original signal into frequency resolution and time resolution contents. For this purpose, a cutting window will be used. This window is known as "Mother Wavelet". The problem here is that cutting the signal corresponds to a convolution between the signal and the cutting window. The signal will convolve with the specified filter coefficients and gives the required frequency information. The decomposition of the image using 2-level DWT is as shown in figure.1.

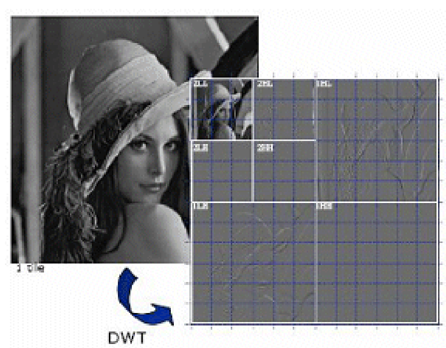

Figure.1. Decomposition of Image [1]

2. WAVELET TRANSFORM

2.1 Discrete Wavelet Transform

The discrete wavelets transform (DWT), which transforms a discrete time signal to a discrete wavelet representation. The first step is to discretize the wavelet parameters, which reduce the previously continuous basis set of wavelets to a discrete and orthogonal / orthonormal set of basis wavelets.

$$\psi_{m,n}(t) = 2^{-m/2}\psi(2^{-m}t - n)\text{; m, n} \in Z \text{ such that } -\infty < \text{m, n} < \infty \quad \text{-------} \quad (3.1)$$

The 1-D DWT is given as the inner product of the signal x(t) being transformed with each of the discrete basis functions.

$$W_{m,n} = \left\langle x(t), \psi_{m,n}(t) \right\rangle; \text{ m, n} \in Z; \text{ m, n} \in Z \quad \text{------} \quad (3.2)$$

The 1-D inverse DWT is given as:

$$x(t) = \sum_{m}\sum_{n} W_{m,n}\psi_{m,n}(t)\text{ ; m, n} \in Z \quad \text{------} \quad (3.3)$$

Discrete signal is passed through a lowpass and highpass filters H and G, then down sampled by a factor of 2, constituting one level of transform. The inverse transform is obtained by up sampling by a factor of 2 and then using the reconstruction filters H' and G', which in most instances are the filters H and G reversed.

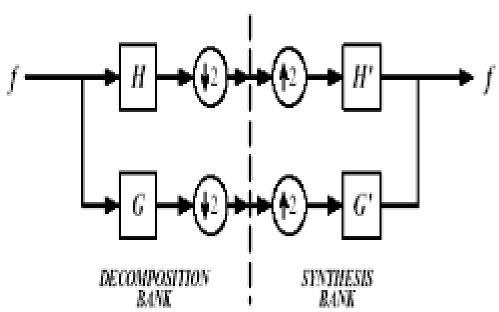

Fig 2 Perfect reconstruction filter bank for used for 1-D DWT

2.2 Two Dimensional Discrete Wavelet Transform

The 1-D DWT can be extended to 2-D transform using separable wavelet filters. With separable filters, applying a 1-D transform to all the rows of the input and then repeating on all of the columns can compute the 2-D transform. When one-level 2-D DWT is applied to an image, four transform coefficient sets are created. As depicted in Figure 3, the four sets are LL, HL, LH, and HH, where the first letter corresponds to applying either a low pass or high pass filter to the rows, and the second letter refers to the filter applied to the columns.

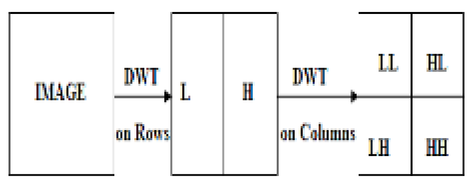

Fig 3 Level one 2-D DWT applied on an image

The Two-Dimensional DWT (2D-DWT) converts images from spatial domain to frequency domain. At each level of the wavelet decomposition, each column of an image is first transformed using a 1D vertical analysis filter-bank. The same filter-bank is then applied horizontally to each row of the filtered and subsampled data. One-level of wavelet decomposition produces four filtered and subsampled images, referred to as sub bands.

We can use multiple levels of wavelet transforms to concentrate data energy in the lowest sampled bands. Specifically, the LL subband can be transformed again to form LL2, HL2, LH2, and HH2 subbands, producing a two-level wavelet transform. An (R-1) level wavelet decomposition is associated with R resolution levels numbered from 0 to (R-1), with 0 and (R-1) corresponding to the coarsest and finest resolutions.

2.3 Reconstruction Filters

The filtering part of the reconstruction process also bears some discussion, because it is the choice of filters that is crucial in achieving perfect reconstruction of the original signal. The down sampling of the signal components performed during the decomposition phase introduces a distortion called aliasing. It turns out that by carefully choosing filters for the decomposition and reconstruction phases that are closely related (but not identical), we can “cancel out” the effects of aliasing.The low- and high pass decomposition filters (L and H), together with their associated reconstruction filters (L' and H'), form a system of what is called quadrature mirror filters:

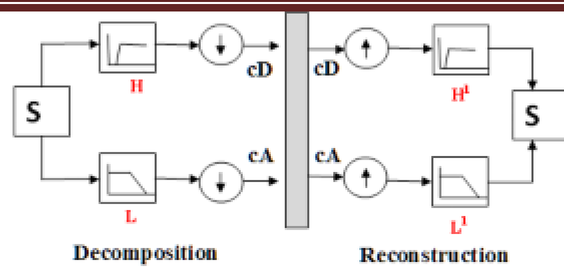

Fig 4 Reconstruction Filters

2.4 Reconstructing Approximations and Details

We have seen that it is possible to reconstruct our original signal from the coefficients of the approximations and
details.

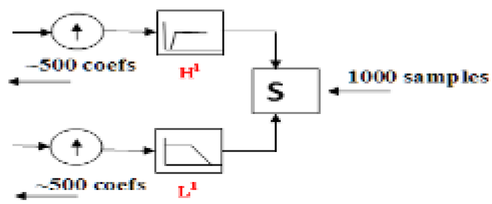

Fig 5 Reconstruction of original signal

It is also possible to reconstruct the approximations and details themselves from their coefficient vectors. As an example, let’s consider how we would reconstruct the first-level approximation A1 from the coefficient vector cA1. We pass the coefficient vector cA1 through the same process we used to reconstruct the original signal. However, instead of combining it with the level-one detail cD1, we feed in a vector of zeros in place of the detail coefficients vector: The process yields a reconstructed approximation A1, which has the same length as the original signal S and which is a real approximation of it. Similarly, we can reconstruct the first-level detail D1, using the analogous process: The reconstructed details and approximations are true constituents of the original signal. In fact, we find when we combine them that:

$A1 + D1 = S$

Note that the coefficient vectors cA1 and cD1—because they were produced by down sampling and are only half the length of the original signal — cannot directly be combined to reproduce the signal. It is necessary to reconstruct the approximations and details before combining them. Extending this technique to the components of a multilevel analysis, we find that similar relationships hold for all the reconstructed signal constituents. That is, there are several ways to reassemble the original signal:

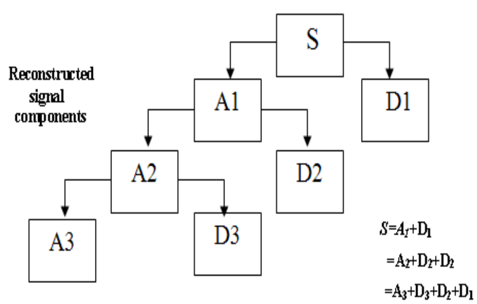

Fig 6 Reconstructed signal components

3. PERFORMANCE COMPUTATION

3.1 QUALITY MEASURES FOR IMAGE

The Quality of the reconstructed image is measured in terms of mean square error (MSE) and peak signal to noise ratio (PSNR).

3.2 MEAN SQUARE ERROR (MSE)

The MSE is often called reconstruction error variance. The MSE between the original image f and the reconstructed image g at decoder is defined as:

$$MSE = \sigma_q^2 = \frac{1}{N} \sum_{j,k} (f[j,k] - g[j,k])^2$$

Where the sum over j, k denotes the sum over all pixels in the image and N is the number of pixels in each image.

3.3 PEAK SIGNAL TO NOISE RATIO (PSNR)

The peak signal-to-noise ratio is defined as the ratio between signal variance and reconstruction error variance. The PSNR between two images having 8 bits per pixel in terms of decibels (dBs). Generally when PSNR is 40 dB or greater, then the original and the reconstructed images are virtually indistinguishable by human eyes.

3.4 COMPRESSION RATIO (CR)

The compression ratio gives how well the image has been compressed. It is defined has

$$CR = \frac{\text{No of bits in original image}}{\text{No of bits in compressed image}} \Longrightarrow 4.3$$

4. Matlab Implementation

In this paper, a software reference model is developed to realize image compression using DWT.The purpose of software reference model is that it helps in understanding the image properties after and before compression and decompression. It also helps in understanding the wavelet filter properties.

4.1 Original Image

Original Image

Fig 7 Input image

Fig 7 shows the original image which is taken for the image compression using HAAR Wavelet transform. DWT is applied to this image.

4.2 Decomposition of Input Image into Levels

Fig 8 shows the Decomposition of the original image into Approximation, Horizontal, Vertical and Diagonal details of level 2.

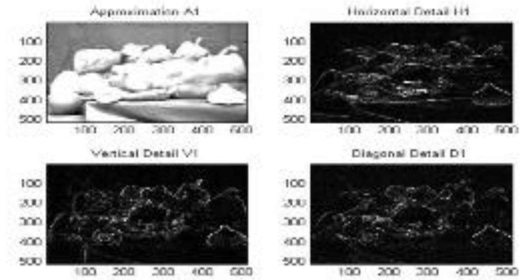

Fig 8 Decomposition level 2

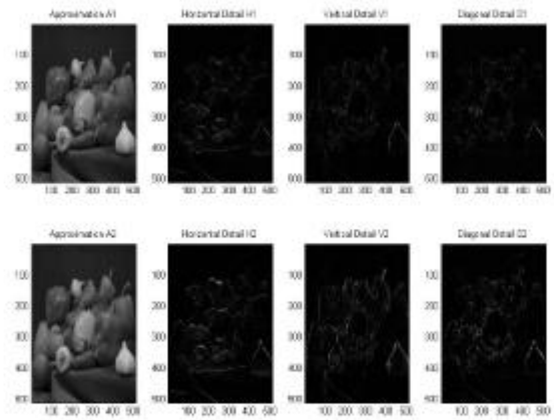

Fig 9 Decomposition level 3

Fig 9 shows the Decomposition of the original image into Approximation, Horizontal, Vertical and Diagonal details of level 3.

Decomposition into Low & High

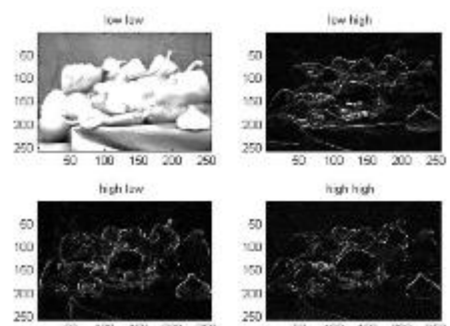

Fig 10 Decomposition into LL, LH,HL,HH

DB2 wavelet achieves better compression Fig 10 is the output obtained when discrete wavelet transform is applied to the original image. Here the decomposition is done by subbanding the original input image into LL, LH, HL, HH components.

4.3 Original Image Vs Re constructed Image

Fig 11 shows the output obtained after reconstruction of the image and the original image.

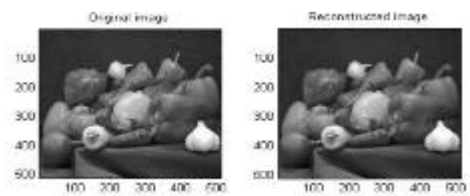

Fig 11 Original image Vs Reconstructed Image

4.4 Comparison of Performance of different Wavelets

Table 1 Comparison of Performance of different Wavelets

BITS PER PIXEL	HAAR			BI-ORTHOGONAL 4.4			DB-2		
	CR	MSE	PSNR	CR	MSE	PSNR	CR	MSE	PSNR
0.25	3.1364	11.464	37.537	4.234	3065.7	13.26	3.4567	629.577	20.14
0.5	4.452	45.492	21.527	3.746	3405.8	12.808	5.345	402.446	22.08
0.1	3.234	691.005	19.736	4.726	3478.9	12.72	5.293	237.86	24.36

5. CONCLUSION & FUTURESCOPE

5.1 CONCLUSION

The performances of various wavelets for image compression using DWT are computed. Input image is processed using DWT and IDWT algorithm. Decomposed sub-band components are selected to achieve different compression ratios. In this case, bits per pixels are used to express the compressed data. Biorthogonal, Haar and DB2 wavelets have been used to compare the performances of DWT-IDWT.MSE, Maximum error and PSNR are the three parameters that are used to analyse the DWT performances. The results of the same are shown in Table 5.5 From the results obtained, it is found that and also reconstruction is better in terms of MSE and PSNR.

5.2 FUTURE SCOPE

Peak Signal to Noise Ratio is to be improved. Through HDL language this will be implemented for low memory requirements and for low power consumptions

6. REFERENCES

[1] M. Nagabushanam, Cyril Prasanna Raj , S. Ramachandran "*Design and FPGA Implementation of Modified Distributive Arithmetic Based DWT – IDWT Processor for Image Compression*" ©20 11 IEEE

[2] David S. Taubman, Michael W. Marcellin – "*JPEG 2000 – Image compression, fundamentals, standards and practice*", Kluwer academic publishers, second printing – 2002.