DESIGN OF LINEAR PRECODER TO MAXIMIZE SUMRATE USING UPLINK

AND DOWNLINK DUALITY

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ABSTRACT: This paper addresses the joint transmit and receive optimization and linear processing for multiuser multiple-input, multiple output (MIMO) downlink. Each user may receive multiple data streams. Our design is to maximize the sum throughput under power constraint is equivalent to minimizing the product of mean square error matrix determinants(PDetMSE).Other one is to minimize the total transmit power with sum rate requirement. A scalar version of mean square error is considered to solve the problem. The problem is product of mean square error does not provide efficient solution. Once the optimal structure of the transmit-receive processing is known, the design problem simplifies and can be formulated within the powerful framework of convex optimization theory, in which a great number of interesting design criteria can be easily accommodated and efficiently solved even though closed-form expressions may not exist. we devise iterative algorithm where each iteration contains the optimization of uplink power, uplink receive filter, downlink receive filter.

For these problem a new power allocation schemes are derived, which can be devised as Differential evolution(DE) involving mean square error. Simulations show that proposed algorithm are proved to be convergent and outperform existing linear methods.

Keywords
Multiuser MIMO systems, transceiver design, sum-rate maximization , Differential evolution algrothim

1.INTRODUTION
THE benefits of using multiple antennas for wireless communication systems are well known. When antenna arrays are present at the transmitter and/or receiver, multiple input, multiple-output (MIMO) techniques can utilize the spatial dimension to yield improved reliability, increased data rates, and the spatial separation of users. In this paper, the methods we propose will focus on exploiting all of these features, with the goal of maximizing the sum data rate achieved in the MIMO multiuser downlink.The achievable capacity region of multiuser MIMO broadcast channels is characterized in an information theoretic context, e.g., [1], [2]. Schemes under the assumption of 'dirty paper precoding' were proposed in [3], . The sum-rate optimality of DPC was generalized to an arbitrary number of multi antenna receivers using the notions of game theory and uplink-downlink duality [4]; this duality is employed in[5] to derive iterative solutions that find the sum capacity.DPC precoding strategy has proven to be a difficult problem.

Existing solutions, which are largely based on Tomlinson-Harashima precoding (THP) [6], incur high complexity due to their nonlinear nature and the combinatorial problem of user order selection.Linear precoding provides an alternative approach for transmission in the MIMO downlink, trading off a reduction in precoder complexity for suboptimal performance. Orthogonalization based schemes use zero forcing (ZF) and block diagonalization (BD) to transform the multiuser downlink into parallel single-user systems.Thus, zero-forcing is not a capacity-achieving strategy. A water filling power allocation can then be used to

allocate powers to each of the users [7]. It is also possible to improve the sum rate achieved with ZF and BD by including user or antenna selection in the precoder design. In this paper, we consider the optimal formulation for sum rate maximization under linear precoding. Much of the existing literature on linear precoding for multiuser MIMO systems focuses on minimizing the sum of mean squared errors (SMSE) between the transmitted and received signals under a sum power constraint [8]–[9]. An important recurring theme in most of these papers is the use of an uplink-downlink duality for both MSE and signal-to interference-plus-noise ratio (SINR) introduced in [10] for the single receive antenna case and extended to the MIMO case in [11].

Linear precoding approaches an iterative method for direct optimization of the sum rate, while [13] and [14] exploit the SINR uplink-downlink duality of [10], [11]. In [16][17] two similar algorithms were independently proposed to minimize the product of the mean squared errors (PMSE) in the multiuser MIMO downlink; these papers showed that the PMSE minimization problem is equivalent to the direct sum rate maximization proposed in [14]. The work of [15] was motivated by the equivalence relationship developed between the single user minimum MSE (MMSE) and mutual information in [18]. Each of the approaches in [12] yields a suboptimal solution, as the resulting solutions converge only to a local optimum, if at all.

It has been shown that with linear processing the above problems are non-convex, thus known algorithms for determinant maximization can not be applied[19]. Here, we propose iterative algorithms to jointly optimize the transmit powers, transmit and receive filters, which are suitable for both uplink and downlink transmission, based on the uplink/downlink duality[20].

Notation: Lower case italics, e.g., x, represent scalars while lower case boldface type is used for vectors (e.g., $\mathbf{x}$). Upper case italics, e.g., N , are used for constants and upper case boldface represents matrices, e.g., $\mathbf{X}$. Entries in vectors and matrices are denoted as $[\mathbf{x}]_i$ and $[\mathbf{X}]_{i,j}$ respectively. The superscripts T and H denote the transpose and

Hermitian operators. E[.] represents the statistical expectation operator while $\mathbf{I}_N$ is the $N \square .N$ identity matrix. tr [.] and det are the trace and determinant operators. $\mathbf{x}_1$ and $\mathbf{x}_2$ denote the 1- norm (sum of entries) and Euclidean norm. diag($\mathbf{x}$) represents the diagonal matrix formed using the entries in vector $\mathbf{x}$.

2.SYSTEM MODEL
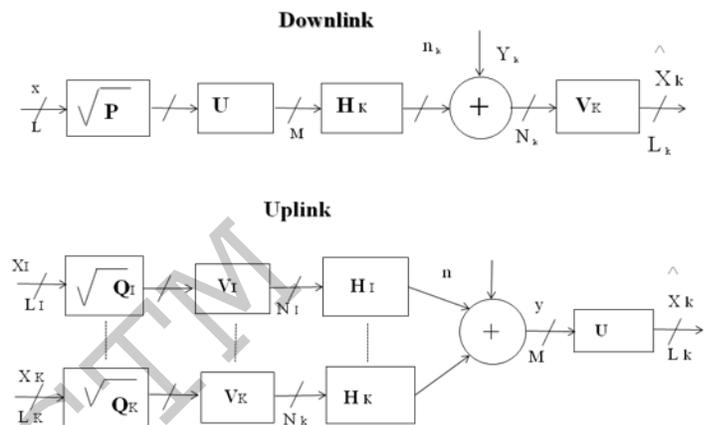
We consider a standard multiuser MIMO downlink model with N_T transmit antennas at base station and K decentralized receivers, each with N_{RK} receive antennas. Assume that independent unit- power symbols $d=[d_1,.....d_K]^T$ with $E\{dd^H\}=I$, are transmitted, where $d_k \in C^{Mk \times 1}$ is the data vector to be transmitted to a Kth mobile. The total number of transmit data streams is $N_d = \sum_{k=1}^{K} M_k$ Zero mean white Gaussian noise is denoted by n=[n1,……,nk] $\sim N(0, \sigma^2_n I)$. These transmitter precoder matrix U = [U_1,. . . ,U_K] $\in C^{NT \times Nd}$ .The downlink transmit power vector for user k be pk = $[p_{k1}, . . . , p_{Nd}]^T$ with P = diag{p}. .The channel matrix is H^H, with H = [H_1, . . . ,H_K] $C^{NT \times NRk}$ .The uplink transmit power vector for user k be qk =[q_{k1}, . . . , $q_N d$ $]^T$ with Q = diag{q}. Based on this model, user k receives a length-N_k vector

$$Y_k = H_k^{\ H} UP_k + n_k \ , \qquad (1)$$

The global receive filter A^H is a block diagonal matrix of dimension $M_k \times N_{RK}$,

$A=$ diag$[A_1, \cdots, A_K]$, where each $A_k =[A_{k1}, \ldots, A_{Nd}]$. Estimate signal

$$\hat{S}_k^{DL} = A_k^H H_k^H U\sqrt{P}\, s_k + A_k^H n_k \quad ,(2)$$

where the superscript DL indicates the downlink. The MSE matrix for user k in the downlink under these general precoder and decoder matrices can be written as

$$E_k^{DL}= V^H_k\, H_k{}^H\, UP_K\, U^H H_k V_k + \sigma^2\, V^H_k\, V_k - V^H_k\, H_k{}^H\, U_k P_K - V_k\, H_k{}^H\, U_k{}^H P_K + I_{Lk} \; ,$$

2.1 Product of MSE matrix determinants:

In this section, we show that an MSE-based formulation using joint processing of all streams leads to an equivalent optimal formulation of the rate maximization problem under linear processing. First, consider the linear MMSE decoder for user k, A_k

$$A_k =(H^H_k UP_k U^H_k H_k+\sigma^2 I)^{-1} H_k^H UP_k \; , \qquad (3)$$

When using this matrix as the receiver in (1), the downlink MSE matrix for user k in can be simplified as

$$E_k^{DL} =I_k - P_K U^H_k H_k J_k^{-1} H_k{}^H U_k, \qquad (4)$$

where $Jk = H^H_k U P_k U^H H_k + \sigma^2 I$ and $R_{N+I,k} = J_k - H^H_k U_k P_k U^H_k H_k$ are the received signal covariance matrix and the noise-plus-interference covariance matrix at user k, respectively.

With this result, we can see that under MMSE reception using A_k as defined in (3), minimizing the determinant of the MSE matrix E^{DL}_k is equivalent to maximizing the achievable rate for user k. It follows that minimizing the product of MSE matrix determinants over all users is equivalent to sum rate maximization,

$$\min\prod_{k=1}^{K}\det E{DL} \equiv \min\sum_{k=1}^{K}\log\det E{DL}$$
$$\equiv \min\sum_{k=1}^{K} R\, LP \qquad (5)$$

The covariance matrices J_k and $R_{N+I,k}$ in the MSE matrix E_k are each functions of all precoder and power allocation matrices. As such, finding U and P jointly or finding only the power allocation P for a fixed U are both non-convex problems and are just as difficult to solve as the rate maximization problem. While it is well accepted that the power allocation sub problem in PMSE minimization is non-convex [18], recent work [17]has shown that the optimal power allocation can be found by formulating the sub problem as a Differential Evolution algorithm problem [19].

3. OPTIMIZATION ALGORTHIM

3.1 Uplink and Downlink MMSE Receive Filters:
In this subsection, we give a short review of the uplink and downlink MMSE receive filters with fixed transmit filters in both links. For convenience, we define

$$\tilde{U}_k=U_k\beta_k=(HAPV^H H^H+\sigma^2_n I)^{-1}H_k A_k P_k, \qquad (6)$$

The diagonal matrix β_k contains the column norms of $\tilde{U}_k$. Similarly, for the downlink channel, the MMSE receive filters are given as $T_K Q_K^{-1/2}$, where

$$\tilde{A}_k=A_k\beta_k=(H_k^H UQU^H H_k+\sigma^2_n I)^{-1}H_k^H U_k Q_k, \qquad (7)$$

The diagonal matrix βk contains the column norms of $\tilde{A}k$.

3.2 Optimization in Virtual Downlink Channel:
Defining diagonal matrix$[D]= \beta_i u^H_i Ht^H_i t_i H^H u_i - 2Re\{u^H_i H t_i\}+1$ and a matrix
$$[\Psi]ij=\begin{cases} u^H_i t_j t^H_j H^H u_i, & i\neq j \\ 0, & i=j \end{cases}$$

According to MSE duality, we know that the same MSE values us in the uplink can be achieved in the downlink, with appositive power allocation

$$p = \sigma_n^2(\varepsilon - D - \beta^2 \Psi^T)^{-1} \beta^2\, 1_{Nd} \; , \qquad (8)$$

With the downlink power allocation=diag$\{q\}$, and transmit filter U, the MSE can be minimized by downlink MMSE receive filters $A_k\beta_k Q_k^{-1/2}$, where A_k, β_k are computed with (7).Additionally, again in the uplink, the power allocation first ensures the same performance as in the downlink, which can be obtained by

$$q = \sigma_n$$

3.3 Optimization in Virtual Uplink Channel:

Under an assumption of minimum Euclidean distance decoding and Gaussian signaling, there is a one-to-one monotonic relationship between achievable rate (in bits per channel use) and SINR.

$$R_i = \log_2(1+SINR),$$

The power optimization problem can be formulated as a Differential Evolution algorithm problem

$$\min_q \prod_{i=1}^{Nd}\{q - 1[(D + \beta 2\Psi)q + \sigma 2\beta 2]i\}wi \qquad (10)$$

DE ALGORTHIM

The DE algorithm is a population based algorithm like genetic algorithms using the similar operators; crossover, mutation and selection. The main difference in constructing better solutions is that genetic algorithms rely on crossover while DE relies on mutation operation. This main operation is based on the differences of randomly sampled pairs of solutions in the population. The algorithm uses mutation operation as a search mechanism and selection operation to direct the search toward the prospective regions in the search space. The DE algorithm also uses a non-uniform crossover that can take child vector parameters from one parent more often than it does from others. By using the components of the existing population members to construct trial vectors, the recombination (crossover) operator efficiently shuffles information about successful combinations, enabling the search for a better solution space.

An optimization task consisting of D parameters can be represented by a D-dimensional vector. In DE, a population of NP solution vectors is randomly created at the start. This population is successfully improved by applying mutation, crossover and selection operators.

The main steps of the DE algorithm is given below:

a. Initialization
b. Evaluation
c. Repeat
d. Mutation
e. Selection
f. Until the required accuracy met

Algroithm:

1. Initialize $[U_t, S_t, V_k] = svd(H_k), A_k = V_k,$ $Q = Q_{max}/K/N_r$ and maximal number of iterations n_{max}.

2. Compute uplink receive filters U_k and β_k, with (7).

3. repeat

4. $n \leftarrow n+1$.

5. Uplink Channel:
   a. For given $A_k^{(n-1)}$, $U_k^{(n-1)}$, and $\beta_k^{(n-1)}$, find optimum power allocation q by solving (9).
   b. Update $U_k^{(n)}$, $\beta_k^{(n)}$, with (7).

6. Downlink Channel:
   a. Compute $P^{(n)}$, with (10).
   b. Update $A_k^{(n)}$, $\beta_k^{(n)}$, with(8).

7. Uplink Channel:
   a. Compute $Q^{(n)}$, with (11).
   b. Update $U_k^{(n)}$, $\beta_k^{(n)}$, with(7).

8. Until required accuracy is reached.

IV. SIMULATION RESULTS

The PMSE and PDetMSE algorithms do not require the explicit selection of *Nd*; rather, this parameter is determined implicitly by the power allocation. However, we can force the PMSE algorithm to allocate a maximum number of substreams *Nd* to each user to gain further insight into its behaviour. In Fig. 5, the number of streams in the *Nd* = 4 system described below is varied from $x1 = x2 = 2$ to $x1 = 3$ and $x2 = 1$. The achievable sum rate in this system decreases in the latter case, as the asymmetric stream allocation does not correspond to the symmetric channel configuration.

MIMO (4,2x2)

POWER MINIMIZATION FOR TX =4,USER=2,REV=2
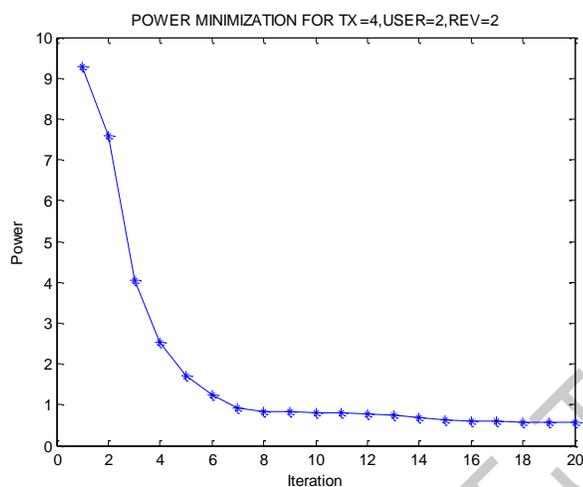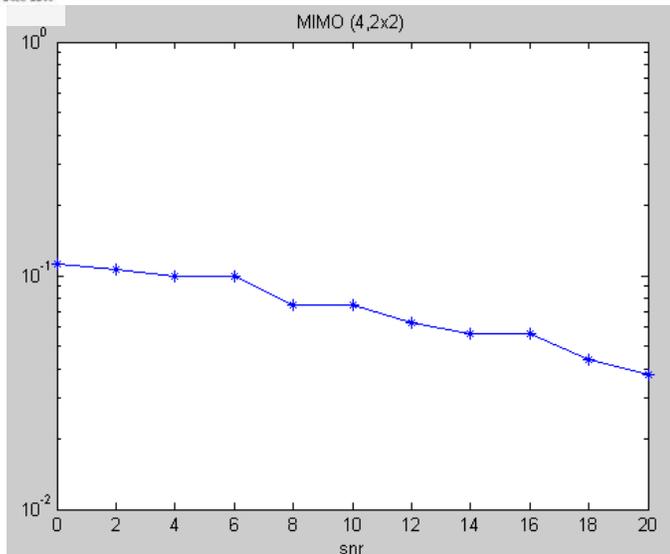
In this case, user 2 is restricted to only a single data stream, and thus can not take full advantage of good channel realizations. If the goal is always, maximizing the sum rate, the users should be allocated the maximum number of data streams in as balanced a manner as possible.

CONCLUSIONS

we have considered the problem of designing a linear precoder to maximize sum throughput in the multiuser MIMO downlink under a sum power constraint. We have compared the maximum achievable sum rate performance of linear precoding schemes to the sum capacity in the general

MIMO downlink. The problem was reformulated in terms of MSE based expressions, and the joint processing solution based on PDetMSE minimization was shown to be theoretically optimal, but computationally infeasible. A suboptimal framework based on scalar processing was then proposed, and an implementation was provided based on PMSE minimization We propose iterative algorithms based on MSE duality and Differential evolution. Unlike the power control by DE with respect to SINR, we propose a new power control strategy by optimizing geometric MSE together with MMSE estimation. The algorithms are proved to be convergent.

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